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DYNAMIC PROCESSES ACCOMPANIED BY MASS TRANSFER—PART 2*

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Аннотация—Рассматриваются результаты новых исследований динамических процессов, сопровождающихся тепломассопереносом. Полученные зависимости служат основой метода исследования энергетических преобразований в потоке.

Газ или парогазовая смесь в процессах испарительного охлаждения (или конденсационного нагрева) рассматривается как особое вещество с переменной единицей массы и преувеличенными термодинамическими свойствами реального газа (сверхреальный газ).

Приводится графическая зависимость частных производных давления и объема смеси по температуре от интенсивности фазового перехода для всех теоретически возможных адиабатных процессов парогазовой смеси (в сдвоенной системе координат). Производится анадиз зависимости, позволивший установить, в частности, пять характерных групп адиабатиых процессов, отличающихся знаком приращени основных параметров.

Находятся выражения для показателя адиабаты k в зависимости от интенсивности фазового перехода ξ. Анализируется завиеимость $k = f(\xi)$ и приводится ее график для всех трех случаев в процессах с испарением.

Полученные таким образом сведения позволяют исследовать преобразование энергии в потоке с тепломассопереносом. Применениз метода показано на примере анакиза зависимости скорости от изменения давления. В частности, показано, что с уменьшением давления скорость может не только возрастать, но и уменьшаться.

NOMENCLATURE

p,	absolute pressure of vapour–gas mixture [N/m^2];
p_{va},	partial pressure of vapour in mixture [N/m^2];
p_s,	saturation pressure at mixture temperature [N/m^2];
V,	volume of mixture per 1 kg of dry gas [m^3/kg of dry gas];
v_{va},	specific vapour volume at its partial pressure and temperature of mixture [m^3/kg];
v'',	specific volume of dry saturated vapour [m^3/kg];
ρ,	mixture density [kg/m^3];
T,	absolute temperature [°K];
φ,	relative humidity;
d_{va},	vapour content [kg/kg of dry gas];
p'_T, V'_T,	partial derivative of pressure and volume containing 1 kg of dry gas with respect to temperature in an adiabatic process;
p'_d, V'_d,	partial derivative of pressure and volume containing 1 kg of dry gas with respect to vapour content in an adiabatic process;
R,	gas constant for mixture containing 1 kg of dry gas [kJ/(kg of dry gas. deg)];
R_g, R_{va},	specific gas constants for gas and vapour [kJ/kg.deg];
U, I,	internal energy and enthalpy of mixture containing 1 kg of dry gas [kJ/kg of dry gas];
u_{va}, i_{va},	internal energy and enthalpy of vapour at its partial pressure and temperature of mixture [kJ/kg];
C_u, C_i,	temperature coefficients of intrinsic energy and enthalpy of mixture per kg of dry gas [kJ/(kg of dry gas. deg)];
C_v, C_p,	isochoric and isobaric heat capacities of mixture containing 1 kg of dry gas [kJ/kg of dry gas];
$c_{v,g}$, $c_{v,va}$,	specific isochoric heat capacities of gas and vapour, [kJ/kg deg];

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* Part 1, see [4].

$c_{p,g}$, c_{pva},	specific isobaric heat capacities of gas and vapour [kJ/kg deg];
k_1, k_2, k_3,	adiabatic exponents in processes with phase transition, heat and mass transfer with independent state parameters pV, TV, T_p, respectively;
ξ,	phase transition rate with respect to temperature [kg/(kg of dry gas. deg];
ω,	phase transition rate with respect to volume [kg/m^3];
q,	process heat [kJ/kg of dry gas];
w,	velocity [m/s].

THERMODYNAMIC phenomena which result from heat and mass transfer in transpiration cooling of gas flow have long attracted the attention of researches. Vulis [1] was engaged in research on these problems back in the early forties. His studies made it possible to establish the influence of various factors upon velocity of gas flow and its state parameters. Consideration was given to the effects of heat exchange, friction, gas flow rate and channel cross section, delivery or withdrawal of work as well as various combinations of these factors.

In particular, the combined effect of heat and mass transfer observed in liquid evaporation or vapour condensation in a gas flow was investigated. The results were reported by the author in several papers and set forth later in a very informative and extensive monograph [1].

Recently research on these problems was started in the U.S.A., where Shapiro and his coworkers of Massachusetts Institute of Technology conducted not only theoretical, but also experimental studies. The results of their studies were published in [2]. Similar experiments were performed in the Soviet Union.

In the theoretical studies conducted, the changes in the states of air and injected liquid were considered separately. With this approach to the investigated problem, certain important features of the processes remained obscure or were not sufficiently clarified.

The method of analysis presented in this paper is based on the reasoning set forth below.

The choice of a unit of mixture quantity or a mass unit is the determining factor in energy research. A variable mass of vapour–gas mixture per 1 kg of dry gas has been adopted as a unit. The liquid is one of the constituents of the environment, and thermal phenomena due to the phase transition are internal phenomena. The mass changes only in connection with phase transition; it controls the phase transition rate and, thus, the caloric properties of the mixture. For example, the adiabatic exponent and temperature coefficients of enthalpy and internal energy ($C_i = \mathrm{d}I/\mathrm{d}T$ and $C_u = \mathrm{d}U/\mathrm{d}T$) ranges within $-\infty$ and $+\infty$ depending on the nature of the process and differ substantially during the process; the quantity $pV/T = R$ (where $R = R_g + d_{va}R_{va}$) is essentially variable.

These quantities are variable for any real gas, but in the considered case under the conditions mentioned they change within extremely wide ranges. Thus, we analyse a certain conventional substance which has hypertrophic thermodynamic properties of a real gas, and in this sense it may be called super-real gas.

The mass-transfer rate as well as the phase transition rate is expressed as a vapour-content derivative with respect to temperature (similar to heat capacity).

$$\xi = \left(\frac{\partial d_{va}}{\partial T}\right)_{dq=0} \tag{1}$$

The processes of transpiration cooling involve usually no essential heat transfer with the surrounding medium and therefore may be considered as adiabatic. There is an infinite number of adiabatic processes with different rates of phase transition. In a particular case, they may take place with a single constant parameter.

Particular cases of such processes have been investigated earlier [3]. Of considerable interest is the generality of their laws valid not only for a stationary gas, but also for gas flow. The derivation of formulae for energy conversions in adiabatic gas flow with transpiration cooling makes it possible not only to discover certain complicated thermodynamic gas phenomena, but also to explain their physical significance.

This paper presents a method of analysis, its thermodynamic fundamentals and examples of application. With this in view, the research was carried out under the following simplifying assumptions:

1. The gas and vapour components of a mixture are assumed to be perfect gases.
2. The fluid was injected at zero value of enthalpy and entropy, i.e. $i_{0,va} = S_{0,va} = 0$ is assumed.
3. The irreversibility due to evaporation of liquid in an unsaturated gas and mechanical irreversibility of the process are neglected.
4. The injection and evaporation of liquid (or condensation of vapour) is assumed to be uniform over the whole volume and adequate for the prescribed rate of phase transition.

To derive general equations for adiabatic processes involving transpiration cooling, we make use of the differential equations and formulae for heat capacities given in [4].

From the differential equations, the following expressions can be obtained for adiabatic processes:

$$p'_T = \left(\frac{\partial p}{\partial T}\right)_{dq=0} = \frac{C_i}{V};$$

$$V'_T = \left(\frac{\partial V}{\partial T}\right)_{dq=0} = -\frac{C_u}{p} \qquad (2)$$

$$p'_d = \left(\frac{\partial p}{\partial d_{va}}\right)_{dq=0} = \frac{C_i}{V\xi};$$

$$V'_d = \left(\frac{\partial V}{\partial d_{va}}\right)_{dq=0} = -\frac{C_u}{p\xi} \qquad (3)$$

where C_i and C_u are temperature coefficients of enthalpy and internal energy determined from the formulae:

$$C_i = C_p + i_{va}\xi; \qquad C_u = C_v + u_{va}\xi. \qquad (4)$$

C_p and C_v are isobaric and isochoric heat capacities of the mixture per kg of dry gas in the processes involving no phase transition.

$$C_p = C_{p,g} + d_{va}C_{p,va}; \qquad C_v = C_{v,g} + d_{va}c_{v,va}.$$

From the formulae for heat capacities in [4] one can determine the rate of phase transition in particular adiabatic processes occurring with one of the following parameters remaining constant: T, p, V, d_{va}. For the processes concerned it takes on the following values respectively:

$$\xi_T = \pm\infty, \qquad \xi_p = -\frac{C_p}{i_{va}}, \qquad \xi_v = -\frac{C_v}{u_{va}},$$

$$\xi_d = 0. \qquad (5)$$

Examination of expressions (2–5) enables us to reveal some specific properties of vapour–gas mixtures and certain phenomena arising from these properties.

Figure 1 shows partial derivatives of the mixture pressure and volume with respect to temperature plotted against the rate of phase transition from formulae (2). The relationship is presented for all adiabatic processes theoretically possible in a vapour and gas mixture under the following conditions: mixture–humid air, $p = 2{\cdot}25 \,.\, 10^5$ N/m², $t = 140$°C, $d_{va} = 0{\cdot}02$ kg/kg dry air.

The rate of phase transition from formula (1) assumes positive values in two cases: with evaporation involving an increase of temperature $dd_{va} > 0$ and $dT > 0$ and in condensation with a decrease in temperature $dd_{va} < 0$ and $dT < 0$. Negative values occur in the cases when dd_{va} and dT have opposite signs.

It follows from the aforesaid that the plot should be presented in twin coordinates: the left-hand system corresponds to the processes with decreasing temperature, the right-hand to those involving increasing temperature. Comparing this with equation (1), we conclude that in the left-hand plot the signs of ξ and dd_{va} are opposite and in the right-hand plot they are coincident. Thus the values of ξ between the ordinates of the twin system correspond to the

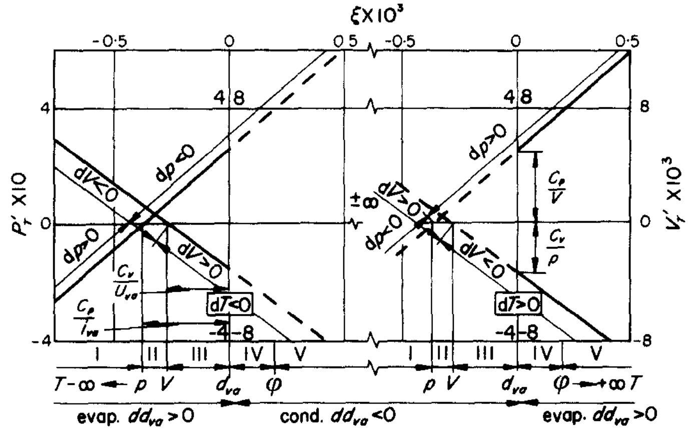

FIG. 1. Partial derivatives p'_T and V'_T vs. rate of phase transition.

processes with vapour condensation, whereas the processes involving evaporation comply with the remaining values of ξ negative in the left-hand and positive in the right-hand plot.

Adiabatic processes with condensation have so far been of little practical interest (except for the cases of $\varphi = 0$ and are not dealt with in this paper. Therefore, when constructing the plot, it is advisable to omit its central part and bring the remaining parts together till their ordinates merge. Considered below are only those parts of the straight lines presented in Fig. 1 which are shown by solid lines.

The intersection point of the straight line p'_T and the abscissa determines the magnitude of evaporation rate ξ_p, at which the process becomes adiabatic–isobaric. In a similar manner, the intersection point of the straight line V'_T and the abscissa characterizes ξ_v for an adiabatic–isochoric process. For an adiabatic–isothermal process, $\xi_T = \pm\infty$. When the negative value of $\mathrm{d}T$ approaches zero, $(\mathrm{d}T = -0)$, then $\xi_T = -\infty$ and, again, if $\mathrm{d}T = +0$ then $\xi_T = +\infty$.

In addition to these three particular processes, Fig. 1 shows two more cases: those of constant vapour content d_{va} and constant relative humidity φ. The particular processes referred to enable one to divide all adiabatic processes into five groups designated by Roman numerals in Fig. 1. Each group has its own specific combination of symbols indicating the growth of the parameters as follows from Fig. 1 and is presented in Table 1.

The dependence of the partial derivative of the mixture pressure and volume with respect to the vapour content upon the rate of evaporation from formulae (3) is presented graphically in Fig. 2 where the dependence of the temperature coefficients C_i and C_u upon ξ is also shown.

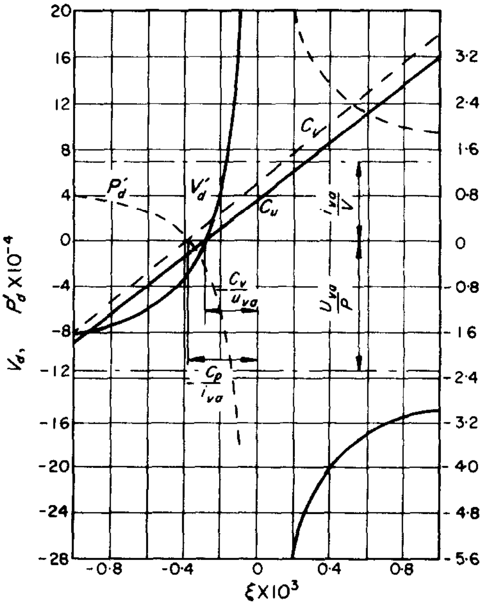

FIG. 2. Partial derivatives p'_d; V'_d and temperature coefficients C_i and C_u vs. rate of evaporation.

Table 1

Group of processes	ξ	$\mathrm{d}d_{va}$	$\mathrm{d}T$	$\mathrm{d}p$	$\mathrm{d}V$	$\mathrm{d}\varphi$
I	$-\infty \Big/ -\frac{C_p}{i_{va}}$	>0	<0	>0	<0	>0
II	$-\frac{C_p}{i_{va}} \Big/ -\frac{C_v}{u_{va}}$	>0	<0	<0	<0	>0
III	$-\frac{C_v}{u_{va}} \Big/ 0$	>0	<0	<0	>0	>0
IV	$0/\xi_\varphi$	>0	>0	>0	<0	>0
V	$\xi_\varphi/+\infty$	>0	>0	>0	<0	<0

where

$$\xi_\varphi = \frac{C_v - \frac{p d_{va}}{\varphi} \cdot \frac{\mathrm{d}v''}{\mathrm{d}T}}{i_{va} - (p - p_S)\frac{v''}{\varphi}}$$

It follows from Figs. 1 and 2 that the processes involving evaporation can only occur with expansion, if the values of the phase transition rates are within the following relatively small limits:

$$-\frac{C_v}{u_{va}} < \xi < 0. \tag{6}$$

For the processes which involve a decrease in pressure, the rate of phase transition assumes values over somewhat wider limits:

$$-\frac{C_p}{i_{va}} < \xi < 0. \tag{7}$$

The limits of these two cases proved to be different. Hence, there is a range of ξ where the signs of dp and dV coincide. Actually, it turns out that in the processes involving evaporation with

$$-\frac{C_p}{i_{va}} < \xi < -\frac{C_v}{u_{va}} \tag{8}$$

d$p < 0$ and d$V < 0$ (for the processes with condensation, it would be d$p > 0$ and d$V > 0$). Consequently, in this range of ξ, the adiabatic index should assume negative values. The adiabatic index has been investigated in more detail.

This made it possible to establish that the adiabatic index takes on various values in the equations comprising different combinations of the thermal parameters (p, V; T, V; T, p) and varies with the rate of evaporation over infinite limits from $-\infty$ to $+\infty$, some of its positive values being attained at $\xi \to \pm\infty$.

To obtain the expressions for calculation of the adiabatic index, we make use of the following relationship for a mixture containing 1 kg of dry gas:

$$\mathrm{d}Q = C_u \mathrm{d}T + p\mathrm{d}V = 0 \ [\text{kJ/kg dry gas}] \tag{9}$$

$$\mathrm{d}Q = C_i \mathrm{d}T - V\mathrm{d}p = 0 \ [\text{kJ/kg dry gas}] \tag{10}$$

$$pV = RT \tag{11}$$

$$C_i - C_u = R + p_{va} \cdot v_{va} \cdot \xi \ [\text{kJ/(kg dry gas-deg)}] \tag{12}$$

From equation (11) we get:

$$dT = (p\,.\,dV + V\,.\,dp - T dR)/R. \qquad (13)$$

Relationships (9–13) and formulae (4) make it possible to obtain an equation for the adiabatic process with transpiration cooling and the expressions for the adiabatic index, making the deductions by three methods different from each other by initial equations and deriving three process equations which correspond to three probable combinations of the varying parameters in each of the three cases concerned.

(1) Initial equation (9). Substituting the value of dT from formula (10) and then making use of equality (12), we find:

$$k_2 \frac{dV}{V} + \frac{dp}{p} - \frac{dR}{R} = 0 \qquad \text{(a)}$$

where

$$k_2 = \frac{C_i - p_{va} v_{va} \xi}{C_u} \qquad (14)$$

is a local adiabatic index which for convenience will be used with subscript 2. (Other expressions designated as k_1 and k_3 will be discussed later).

If k_2 is assumed to be constant within the whole process or a part of it, then, with p and V independent, one can obtain an adiabatic equation from (a):

$$\frac{pV^{k_2}}{R} = \text{const.}$$

The latter enables us to obtain adiabatic equations (where T, V and T, p are variable) with the aid of (11):

$$\left.\begin{aligned} TV^{k_2 - 1} &= \text{const.} \\ T\left(\frac{R}{p}\right)^{(k_2 - 1)/k_2} &= \text{const.} \end{aligned}\right\} \qquad (15)$$

The adiabatic index k_2 in these equations should be calculated from formula (14). It is noteworthy that of the three process equations obtained, only one (where variables T and V are independent) is of the conventional form. The two other equations are more complicated because they incorporate the third variable R.

(2) Initial equation (10). By substituting in it for the temperature differential from (13) and using (12), we obtain

$$k_3 \left(\frac{dV}{V} - \frac{dR}{R}\right) + \frac{dp}{p} = 0 \qquad \text{(b)}$$

where

$$k_3 = \frac{C_i}{C_u + p_{va} v_{va} \xi} \qquad (16)$$

is another local adiabatic index.

If k_3 is assumed to be constant, the adiabatic equation with variables p and V is obtained on integrating (b):

$$p\left(\frac{V}{R}\right)^{k_3} = \text{const.}$$

Two equations given below are obtained in a similar manner:

$$\left.\begin{aligned} T\left(\frac{V}{R}\right)^{k_3 - 1} &= \text{const.} \\ \frac{T}{p^{(k_3 - 1)/k_3}} &= \text{const.} \end{aligned}\right\} \qquad (17)$$

where the value of the adiabatic index k_3 should be determined from (16).

In this case, out of the three equations obtained, the one containing the independent variables T and p is of the usual form, the adiabatic index being expressed through another relationship and thus numerically different from k_2.

(3) The usual adiabatic expressions (15) and (17) with variables T, V and T, p are the initial equations which can be presented as follows:

$$TV^{R/C_u} = \text{const.}; \qquad T_p^{-R/C_i} = \text{const.}$$

If the former is divided by the latter and the (R/C_i)-th root is extracted from both parts of the equality we get:

$$pV_1^k = \text{const.} \qquad (18)$$

where

$$k_i = \frac{C_i}{C_u}. \qquad (19)$$

Substituting from (11) for the volume in one case and for pressure in the other in equality (18) we obtain adiabatic equations incorporating variables T, V and T, p which take the form:

$$RTV^{k_1-1} = \text{const.}$$

$$\frac{RT}{p^{(k_1-1)/k_1}} = \text{const.}$$

Consequently, we obtain only one ordinary adiabatic equation with variables p and V (18), the adiabatic index again being expressed in a new form.

Thus, it follows from the above, that if adiabatic equations incorporating variables T, V; T, p and p, V are of the usual form, the adiabatic indices in these three cases will be different. The adiabatic indices are shown in Table 2.

Table 2 also gives the relationships for the adiabatic index in some particular cases of the adiabatic process. The relationships are readily obtained from appropriate general expressions, if it is taken into consideration that in an adiabatic–isochoric process $C_u = 0$, adiabatic–isobaric $C_i = 0$, adiabatic–isothermic $\xi = \infty$ and in an adiabatic process with constant vapour content $\xi = 0$. The relationships for the adiabatic index κ_φ in a process with constant relative humidity φ are somewhat more difficult to obtain. We shall not, however, discuss the method for obtaining these relationships.

When the rate of phase transition ξ changes, temperature coefficients C_u and C_i change substantially. This produces an effect upon the adiabatic index as is evident from the general expressions for k_1, k_2 and k_3. This relationship is graphically shown in Fig. 3.

The calculation was made for air under the following conditions: $p = 2{\cdot}25 \,.\, 10^5$ N(m^2, $t = 500$°C, $d_{va} = 0{\cdot}02$ kg/kg dry air and at various evaporation rates $\xi = (-1 \div +0{\cdot}6)\,.\,10^{-3}$ kg/kg dry air deg.

In all three cases, the relationship $k = f(\xi)$ is expressed by fractional linear functions plotted as equilateral hyperbolae with asymptotes parallel to the coordinate axes. To determine the asymptotes of such hyperbolae, it is sufficient

Table 2. Adiabatic index

Independent variables	General expressions	For particular adiabatic processes				
		V = const. k_v	p = const. k_p	T = const. k_T	d_{va} = const. k_d	φ = const. k_φ
p, V	$k_1 = \frac{C_i}{C_u}$	∞	0	$\frac{i_{va}}{u_{va}}$	$\frac{C_p}{C_v}$	$\frac{k_d(1+a) - i_{va}/u_{va}(1+b)}{a-b}$
T, V	$k_2 = \frac{C_i - p_{va}V_{va}\xi}{C_u}$	∞	$-\frac{p_{va}V_{va}\xi}{C_u}$	1	$\frac{C_p}{C_v}$	$\frac{k_d(1+a) - (1+b)}{a-b}$
T, p	$k_3 = \frac{C_i}{C_u - p_{va}v_{va}\xi}$	$\frac{C_i}{p_{va}v_{va}\xi}$	0	1	$\frac{C_p}{C_v}$	$\frac{k_d(1+a) - (1+b)}{a - i_{va}/u_{va}\, b}$

Notes:

$$a = \frac{pv_{va}}{u_{va}}; \qquad b = pd_{va}\left(\frac{\partial v_{va}}{\partial T}\right)_\varphi \Big/ C_v$$

Indexes k_{2_p} and k_{3_v} assume negative values.

With φ = const., the process is possible at temperatures not exceeding the critical temperature for steam.

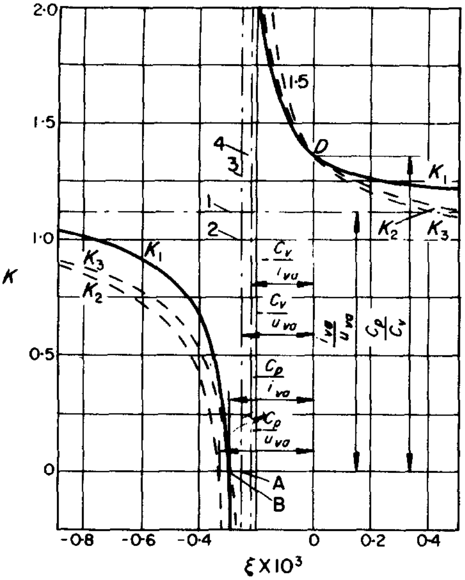

FIG. 3. Values of local adiabatic indices for processes involving evaporation of water injected into humid air (1,2,3,4-asymptotes for k_1; k_2 and k_3; k_1 and k_2; k_3, respectively).

to find their centre coordinates in each of the three cases. Since the hyperbolae equations are known [equations (14, 16, 19)], the coordinates can be found readily with the aid of ordinary mathematical methods. The centre coordinates determined for each hyperbola are given in Table 3.

It is evident from Table 3 and Fig. 3 that $\xi = -C_v/U_{va}$ is the centre abscissa of two hyperbolae relating to k_1 and k_2. It follows from (5) that this rate of phase transition makes the process adiabatic–isochoric. Consequently, k_1 and k_2 in the adiabatic–isochoric process pass through $\pm\infty$. Point A represents the process on the abscissa.

Table 3. Coordinates of hyperbola centre

Centre coordinates	For $k_1 = f_1(\xi)$	For $k_2 = f_2(\xi)$	For $k_3 = f_3(\xi)$
ξ_c	$-\frac{C_v}{u_{va}}$	$-\frac{C_v}{u_{va}}$	$-\frac{C_v}{i_{va}}$
k_c	$\frac{i_{va}}{u_{va}}$	1	1

As shown in Fig. 1, the region on the graph between the asymptote passing through point A and the ordinate (with $\xi = -C_v/u_{va} \div 0$) refers to the processes with expansion. Therefore, the curve sections within this region determine the values of the adiabatic index for expansion processes and those to the right and left of this region give the adiabatic index in the processes with compression.

The curves in Fig. 3 cross the abscissa to the left of point A and, hence, in the processes that involve evaporation of the liquid injected, the adiabatic indices k_1, k_2 and k_3 can assume zero values only in compression processes with a decrease in temperature.

Indices k_1 and k_3 become zero at the same value of ξ, and their respective curves intersect at point B (Fig. 3). To determine the value of ξ for this point, we equate the expressions for k_1 or k_3 from (16) or (19) to zero, finding as a result that $C_1 = 0$. Expressing C_i in compliance with (4),

$$\xi_p = -\frac{C_p}{i_{va}}.$$

The values obtained for C_i and ξ determine the adiabatic–isobaric behaviour of the process and point B represents these values on the abscissa.

The adiabatic index k_2 assumes the zero value, as results from (14) and (4) at

$$C_i - p_{va}v_{va}\,\xi = C_p + u_{va}\xi$$

or for the evaporation rate of

$$\xi = -\frac{C_p}{u_{va}}.$$

All three curves intersect the ordinate in one point D. Since then $\xi = 0$, it follows that $k = C_p/C_v$, i.e. the adiabatic index assumes an ordinary expression for a process without phase transition.

With the rise in temperature, k_2 and k_3 increase, if $\xi < 0$, and they decrease, if $\xi > 0$, while k_1 increases for all the values of ξ whilst point B approaches the ordinate and point D the abscissa.

The graphical relationship given in Fig. 3 is plotted for a gas temperature (500°C) above the critical point of water vapour (374°C) owing to which a physical state characterized by relative humidity is impossible. In this case, the adiabatic processes involving evaporating cooling should be divided into four groups. The ranges of the adiabatic indices for each of the four groups of processes are given in Table 4.

Table 4. Limits of the values of adiabatic index for processes involving evaporation

Group of processes	ξ	k_1	k_2	k_3
I	$-\infty\Big/-\dfrac{C_p}{i_{va}}$	$\dfrac{i_{va}}{u_{va}}\Big/0$	$1/k_{2p}$	$1/0$
II	$-\dfrac{C_p}{i_{va}}\Big/-\dfrac{C_v}{u_{va}}$	$0/-\infty$	$k_{2p}/-\infty$	$0/k_{3v}$
III	$-\dfrac{C_v}{u_{va}}\Big/0$	$+\infty/k_d$	$+\infty/k_d$	k_{3v}/k_d
IV	$0/+\infty$	$k_d\Big/\dfrac{i_{va}}{u_{va}}$	$k_d/1$	$k_d/1$

Note: k_d, k_{2p} and k_{3v} are taken from Table 2.

Only group III contains the processes with expansion. The compression processes characteristic of the first two groups differ from those falling into group IV in the sign of the temperature gain. The processes under groups I and II occur with $\mathrm{d}T > 0$ which may be attributed to a substantial cooling. It is convenient in this case to take the rate of phase transition with respect to the volume as a criterion of the cooling rate.

$$\omega = \left(\frac{\partial d_{va}}{\partial V}\right)_{\mathrm{d}q=0}$$

The processes falling into the first two groups are accompanied by intense cooling

$$[\omega] > \frac{p}{u_{va}}.$$

In the processes of group IV, $\mathrm{d}T > 0$, because the rate of cooling is rather small:

$$[\omega] < \frac{p}{u_{va}}.$$

An isothermal process with which the left- and right-hand sides of the inequalities given above are equal serves as a boundary between the processes falling into groups I and IV. This occurs with $\xi = \pm\infty$.

With the state parameters for which the curves presented in Fig. 3 are plotted, the numerical values of the quantities given in Table 4 are as follows:

$\xi_p = -C_i/i_{va} = -0{\cdot}31\,.\,10^{-3}$ kg/kg dry air deg

$\xi_v = -C_v/u_{va} = -0{\cdot}25\,.\,10^{-3}$ kg/kg dry air deg

$k_{1T} = i_{va}/u_{va} = 1{\cdot}116;\ k_{2p} = -0{\cdot}602;\ k_d = 1{\cdot}38$

$k_{3V} = -2{\cdot}29$; the rate of phase transition with respect to the volume $\omega = -0{\cdot}722$ kg/m^3.

Specified in Table 4 are the processes falling into group II for which the three adiabatic indices assume negative values, k_2 in group I and k_3 in group III being partially negative.

The data obtained on the behaviour of the adiabatic indices of the partial derivatives of the thermal parameters of state and their differentials make it possible to investigate energy conversion both in a stationary gas and in gas flow. In the latter case, it is also necessary to determine the partial derivatives of the flow velocity (or some other function under study) with respect to the pressure, temperature, vapour content or some other parameter regarded as in independent variable and a controlling factor in the investigation.

Assuming the rate of cooling liquid injected to be zero, the following relationship can be obtained ($j = \rho w$ is the flux density):

$$\left(\frac{\partial w}{\partial p}\right)_{\mathrm{d}q=0} = -\frac{1}{j}\left(1 + \frac{w^2\xi}{C_i}\right), \frac{\mathrm{m}^3}{\mathrm{s\,.\,N}}. \quad (20)$$

Figure 4 presents $(\partial w/\partial p)_{\mathrm{d}q=0}$ against the rate of evaporation (for the same conditions as in Fig. 1). The curve is a hyperbola and the centre coordinates are:

$$\left(\frac{\partial w}{\partial p}\right)_c = \left(\frac{\partial w}{\partial p}\right)_{\mathrm{d}q=0} = -\frac{1}{j}\left(1 + \frac{w^2}{i_{va}}\right)$$

$$\xi_c = \left(\frac{\partial d_{va}}{\partial T}\right)_{\mathrm{d}q=0} = -\frac{C_p}{i_{va}}.$$

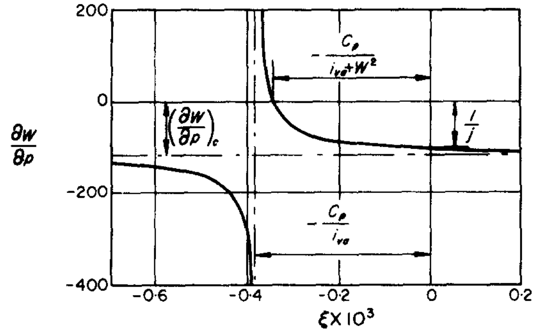

FIG. 4. $(\partial W/\partial p)_{\mathrm{d}q=0}$ vs. evaporation rate.

With ξ increasing from $-C_p/i_{va}$ to 0, that is, in the processes involving a pressure drop (see Fig. 1), the derivative $(\partial w/\partial p)_{\mathrm{d}q=0}$ decreases from $+\infty$ to 0 and then to $-1/j$ which is true for a process without phase transition.

In the case of a pressure drop ($\mathrm{d}p < 0$), the derivative from formula (20) can be not only negative, but also positive. Its transition through 0 occurs, when the evaporation rate reaches a certain critical value

$$\xi_{cr} = -\frac{C_p}{i_{va} + w^2}. \quad (21)$$

If $-C_p/i_{va} < \xi < \xi_{cr}$, then $\mathrm{d}w < 0$. If $\xi_{cr} < \xi < 0$, then $\mathrm{d}W > 0$.

In the case of a pressure rise (both with $-\infty < \xi < -C_p/i_{va}$ and $0 < \xi < +\infty$), $\mathrm{d}w < 0$.

In most cases,

$$[\xi_{cr}] > \frac{C_v}{u_{va}} \quad (22)$$

and is, therefore, within the range of the processes falling into group II. However, it may turn out at high rates that

$$[\xi_{cr}] < \frac{C_v}{u_{va}}. \quad (23)$$

If so, a simultaneous decrease in pressure and velocity will occur in some processes falling into group III as well, being attended not with a decrease, but an increase in the volume of the mixture (see Fig. 1).

Table 5

t °C	$\frac{C_p}{i_{va}} . 10^3$	$\frac{C_v}{u_{va}} . 10^3$	$\xi'_{cr} . 10^3$	$\xi''_{cr} . 10^3$
140	0·382	0·295	−0·350	−0·280
500	0·325	0·268	−0·304	−0·253

ξ' and ξ'' are the critical values of the evaporation rate at flow velocities of 500 m/s and 1000 m/s, respectively.

For instance, it follows from Table 5 in which are given the values calculated for the same conditions as for the diagrams shown earlier, that with the velocity of 500 m/s the inequality (22) and with 1000 m/s the inequality (23) hold true, that is, ξ_{cr} passes through a boundary between groups II and III. This occurs, when the flow velocity attains a certain boundary value which can be determined from

$$\xi_{cr} = -\frac{C_v}{u_{va}}$$

whence, comparing it with formula (21), one can obtain an expression for the velocity at the boundary:

$$w = \sqrt{(ku_{va} - i_{va})}.$$

At temperatures of 140, 500 and 1000°C w is 900,865 and 812 m/s, respectively.

These analytical and graphical relationships make it possible to derive, in addition to the equations given above, also other formulae which are unusual for simple substances.

Hence, it is concluded that:

(1) The local adiabatic index assumes values from $-\infty$ to $+\infty$ depending on the rate of evaporation.

(2) In the adiabatic equations incorporating different independent variables (p, V; T, V; T, p), the adiabatic index takes on different numerical values.

(3) In the processes falling into group II, that is at the evaporation rate of

$$-\frac{C_p}{i_{va}} < \xi < -\frac{C_v}{u_{va}}$$

all three adiabatic indices assume negative values.

(4) These unusual properties in the processes with transpiration cooling result in unexpected phenomena in a fluid flow. For instance, compression can give rise to a pressure drop which can give rise not to an increase, but to a decrease in flow velocity, etc.

(5) The method set forth above allows certain thermodynamic and gas dynamic phenomena in transpiration cooling processes to be investigated and explained, taking into account the effect of these phenomena on the results of the processes concerned.

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Abstract—The results of recent research on dynamic processes with heat and mass transfer are considered.

A gas or a vapour–gas mixture in these processes is considered as a specific substance with a variable mass unit and hypertrophic thermodynamic properties of a real gas (a super-real gas).

A graphical relationship of partial derivatives of mixture pressure and volume with respect to temperature as a function of the rate of phase transition for all adiabatic processes theoretically possible in a vapour–gas mixture is given.

Expressions for the adiabatic exponent as a function of the rate of the phase transition ξ are derived. Relationship $k = f(\xi)$ is analyzed and is plotted for all three cases of processes with evaporation.

The data thus obtained on the behaviour of the adiabatic exponent and partial derivatives of the state thermal parameter j make it possible to investigate energy transformations in a flow with heat and mass transfer.

Application of the method is illustrated by the analysis of velocity against pressure.

Résumé—Les résultats d'une recherche récente sur les processus dynamiques avec transport de chaleur et de masse sont pris en considération.

Un gaz ou un mélange de vapeur et de gaz est considéré dans ces processus comme une substance spécifique avec une unité de masse variable et des propriétés thermodynamiques hypertrophiées d'un gaz réel (gaz sur-réel).

Une relation graphique des dérivées partielles de la pression et du volume du mélange par rapport à la température est donnée en fonction de la vitesse du changement de phase pour tous les processus adiabatiques possibles théoriquement dans un mélange de vapeur et de gaz.

Des expressions sont obtenues pour l'exposant adiabatique en fonction de la vitesse de changement de phase ξ. La relation $k = f(\xi)$ est analysée et tracée dans trois cas de processus avec évaporation.

Les résultats obtenus ainsi sur le comportement de l'exposant adiabatique, et les dérivées partielles des paramètres d'état thermiques rendent possibles l'étude des transformations d'énergie dans un écoulement avec transport de chaleur et de masse.

L'application de la méthode est illustrée par l'analyse de la vitesse en fonction de la pression.

Zusammenfassung—Die Ergebnisse einer kürzlich durchgeführten Untersuchung über dynamische Prozesse mit Wärme- und Stoffübergang werden angegeben.

Ein Gas- oder ein Gasdampfgemisch wird in diesen Prozessen über eine spezifische Substanz mit veränderlicher Masse und überentwickelten thermodynamischen Eigenschaften eines realen Gases betrachtet (ein super-reales Gas). Eine graphische Beziehung wird angegeben für partielle Ableitungen des Gemischdruckes und des Volumens nach der Temperatur als Funktion der Phasenübergangsgeschwindigkeit für alle theoretisch möglichen adiabaten Prozesse in einem Dampf–Gasgemisch.

Der Adiabatenexponent wird als Funktion der Phasenübergangsgeschwindigkeit ξ abgeleitet. Die Bezeichnung $k = f(\xi)$ wird analysiert und ist für alle drei Fälle der Prozesse mit Verdampfung aufgezeichnet.

Die so erhaltenen Werte für das Verhalten des Adiabatenexponenten und der partiellen Ableitung der thermischen Zustandsparameter ermöglichen die Untersuchung in einer Strömung mit Wärme- und Stoffübergan. Die Anwendung der Methode wird gezeigt.